AMERICAN
STATISTICAL ASSOCIATION.

NEW SERIES, No. 94. JUNE, 1911.

# VITAL AND MONETARY LOSSES IN THE UNITED STATES DUE TO TYPHOID FEVER.*

By William O. Mendenhall, *Professor of Mathematics, Earlham College,*
and
Earl W. Castle, *Instructor in Mathematics, University of Michigan.*

## I.

The study outlined in this paper was made under the direction of James W. Glover, Professor of Mathematics and Insurance in the University of Michigan. Its object is twofold: first, to determine the vital and financial loss to a community due to typhoid fever; and second, to give an example of the construction of mortality tables by Sprague's Osculatory Method. This method of construction is treated in a paper by King.† By the method here described it is possible to represent the given data with remarkable accuracy, and to obtain at the same time a smooth graduation. More of detail will be given in the second part of the discussion, which deals with the construction of the mortality table, than in the first part, which concerns the determination of loss due to typhoid fever, since the method of finding the effect of a disease upon a community has been completely illustrated in a paper by Glover.‡

The statistics upon which the mortality tables are based are the enumerated population, aggregate male and female, June 1, 1900, obtained from the reports of the Twelfth Census of the United States, and the Mortality Statistics of the registration states for the calendar year 1900. These registration

* Based on the returns from registration states of the Twelfth Census of the United States.

† George King, "On the Construction of Mortality Tables from Census Returns and Records of Deaths," *Journal of the Institute of Actuaries*, Vol. 42, p. 225.

‡ James W. Glover, "The Monetary Loss in the United States Due to Tuberculosis, Based on the Returns of the Twelfth Census of the United States," *Transactions of the Sixth International Congress on Tuberculosis*, Vol. 5, p. 55.

states were Connecticut, District of Columbia, Indiana, Maine, Massachusetts, Michigan, New Hampshire, New Jersey, New York, Rhode Island, and Vermont. The records of the registration states are used because sufficiently complete and accurate data cannot be obtained for the other states. From the data at our disposal the statement appears to be conservative that the mortality in these states due to typhoid fever is no greater than that throughout the United States. Hence the results obtained on this basis apply to the population of the United States and may be applied in the same way to the population of any state or city where similar information is desired. If in the community in question the death-rate due to typhoid fever be less than the average for this area, these results may be proportionately reduced.

Two sets of life-tables were prepared on the basis of the statistics mentioned above. The first one takes into account deaths from all causes, and so represents the vital condition of the community as it was in 1900. The second table takes into account deaths from all causes except typhoid fever, and so represents the status as it would be in a population subject to all other conditions existing in 1900, but with no deaths from typhoid. The difference between these tables is a quantity which represents the loss due to typhoid mortality, and is the foundation for other computations intended to exhibit this loss from different points of view. The loss here shown is the total loss due to typhoid mortality in a given group living under such conditions that the death-rate due to typhoid is 31.3 per 100,000 of population, the average for the registration states in 1900.

To use these results to measure the minimum community loss due to impure water, we have only to show that this loss exceeds the loss from typhoid mortality in the group above mentioned. In making this comparison it should be first noted that only the cases of typhoid fever which terminate fatally affect the results given, whereas it is estimated by Whipple* that fifteen to eighteen cases occur for every death; it follows that the monetary loss due to morbidity in typhoid, though not considered here, must be extremely great.

* George Whipple, *The Value of Pure Water*, p. 8. New York, 1907.

Second, in finding the total loss to a community which may be prevented by sanitary measures, it should be remembered that the loss from other diseases which spring from the same causes is so great that the typhoid mortality represents less than half * of the total loss due to the conditions which produce typhoid fever.

Third, the loss shown in this paper is due only to deaths which are to occur among the present male population of the community who have attained age twenty, and does not include the losses which the community will sustain due to deaths among those who shall attain this age later.

It is well known that the amount of typhoid fever in a city or village is largely dependent upon the nature of the water supply. However, since there are other means of transmission of the disease, there may be some typhoid even when the water supply is perfect. That due to causes other than impure water is called residual typhoid. Where sanitary conditions are good, an annual death-rate of about 15 per 100,000 seems to represent fairly the residual typhoid, though this is often reduced to 10, and sometimes even less. When this is compared to death-rates of 60 and 100, which were the rates in some cities in 1907 and 1908, or even with the average rate in the total registration area of this country, which was 30.3 in 1907, it becomes apparent that the removable loss is over 15 per 100,000 for the registration area, and becomes in some cities as high as 45 to 85. We believe, in view of what has been stated, that the results finally derived in this paper may be considered minimum loss-rates based on a pure water supply. That is, we may conclude that a community, which with an impure water supply has a typhoid death-rate not less than that in the registration area, is losing through the existence of typhoid in excess of the possible residual an amount as great as that represented by the loss-rates to be given in this discussion.

Although the results show a great saving, the conservative character of the method again appears when the benefit of an improved water supply in various cities is considered. A few

* Whipple, *loc. cit.*, pp. 7–12.

examples may be cited. In the city of Camden, New Jersey,* in the years 1890 to 1899, with the population ranging from 58,313 in 1890 to 75,935 in 1900, there were 297 deaths due to typhoid fever, nearly 30 a year. In 1899 the water supply of the city was changed from the Delaware river to artesian wells. In the years 1900 to 1906 the total number of deaths from typhoid was 37—a little more than five per year,—with the population increasing to 92,458 in 1907.† In Albany, New York,‡ the death-rate per 100,000 due to typhoid fever in years 1894 to 1898 was 104; in years 1900 to 1904 it was 26. Filtered water was introduced into Albany in 1899. In Ithaca, New York,§ for several years prior to 1903 the average death-rate per 100,000 from the same cause was 67.2. Since that time, when filters were introduced, the rates for the years 1904 to 1908 have been 13.7, 6.8, 0, and 6.4, respectively.

When sanitary methods have become universal, the residual typhoid should be still further decreased, since the number of loci of infection will be made smaller.

Often in cities with a good water supply, hospitals receive cases of this disease contracted elsewhere, and the mortality is charged against the city in which the hospital is located, making the residual appear higher than it really is. Since, as has been pointed out, the results obtained in this paper are conservative, even though the residual typhoid cannot be eliminated from the community, it is apparent that any lowering of this residual would increase the monetary gain to the community to figures above those shown in this paper.

The results here obtained are based upon the fundamental figures shown in Table I which are drawn from the census reports.

* Report of Chief Engineer, Camden City Water Department, 1907.
† Police Census.
‡ Whipple, *The Value of Pure Water.*
§ *Bulletin* New York State Department of Health, April, 1908. April, 1909.

TABLE I.

POPULATION AND VITAL STATISTICS, REGISTRATION STATES, TWELFTH CENSUS.

| Age Group. | Total Population June 1, 1900. | Deaths from All Causes, 1900. | Deaths from Typhoid Fever, 1900. | Deaths from All Causes Except Typhoid Fever, 1900. |
|---|---|---|---|---|
| Under 1 | 437,944 | 71,117 | 43 | 71,074 |
| 1– 2 | 394,138 | 16,866 | 52 | 16,814 |
| 2– 3 | 412,353 | 7,439 | 60 | 7,379 |
| 3– 4 | 416,695 | 4,691 | 67 | 4,624 |
| 4– 5 | 411,668 | 3,416 | 58 | 3,358 |
| 5– 9 | 1,984,846 | 9,242 | 347 | 8,895 |
| 10–14 | 1,819,115 | 5,423 | 569 | 4,854 |
| 15–19 | 1,804,950 | 8,746 | 746 | 8,000 |
| 20–24 | 1,905,779 | 13,013 | 890 | 12,123 |
| 25–29 | 1,839,826 | 14,533 | 854 | 13,679 |
| 30–34 | 1,630,050 | 13,991 | 637 | 13,354 |
| 35–39 | 1,474,697 | 14,331 | 479 | 13,852 |
| 40–44 | 1,247,880 | 13,736 | 393 | 13,343 |
| 45–49 | 1,013,403 | 13,416 | 301 | 13,115 |
| 50–54 | 872,741 | 14,886 | 216 | 14,670 |
| 55–59 | 685,469 | 16,101 | 159 | 15,942 |
| 60–64 | 562,777 | 17,988 | 124 | 17,864 |
| 65–69 | 414,450 | 19,568 | 95 | 19,473 |
| 70–74 | 288,229 | 20,154 | 64 | 20,090 |
| 75–79 | 176,031 | 18,477 | 40 | 18,437 |
| 80–84 | 87,093 | 14,053 | 20 | 14,033 |
| 85–89 | 31,558 | 7,418 | 5 | 7,413 |
| 90–94 | 7,989 | 2,622 | 1 | 2,621 |
| 95–99 | 1,576 | 802 | 0 | 802 |

The second column of Table I gives the total population of the registration states at the various ages indicated in the first column. After the fifth year, the ages are taken in groups of five. In the succeeding columns are shown the number of deaths occurring in each age group due to all causes and the number due to typhoid fever. For example, it appears that of the total population in these states on the date of the enumeration, there were 1,839,826 persons whose ages were over twenty-five and under thirty years; that during the calendar year 1900 among persons of these ages there had occurred 14,533 deaths from all causes; that 854 of these were due to typhoid fever, and 13,679 to other causes.

An understatement of deaths may be possible, but as our object is to exhibit the difference between two tables which depend upon the same returns, this understatement will affect our results only in so far as there is a difference in the percentage of understatement between the deaths due to typhoid fever and those due to other causes. The third column of Table II exhibits a mortality table with typhoid present. It was obtained by methods explained later in this paper from the second and third columns of Table I.

With typhoid mortality existing at various ages as exhibited in Table I, a group of 1,000,000 persons aged ten years will have such a life history as illustrated in the third column of Table II. Of this group there will remain living at age twenty, 959,714; at age fifty, 720,505; at age one hundred, 5. By using in the same way the data of the fifth column, Table I, we get the mortality table in the second column of Table II. It appears that the number living at each age beyond ten is greater than at the corresponding age in the former case.

In the fourth column, the difference between the two is tabulated. Thus at age twenty, there would be living 3,468 more people of the original group, if typhoid mortality were eliminated, than under existing conditions with typhoid present in the community. At age fifty-two this difference becomes 10,832, which is the maximum.

These mortality rates and the differences are shown in Table III in the more usual manner, in the form of death-rate per 1,000 per annum.

This death-rate is obtained from Table II by finding the ratio of the number dying in any year, $d_x$, where x denotes the age, to the number living at the beginning of the year, $l_x$, and multiplying this ratio by 1,000. The greatest losses are at the ages from fifteen to thirty-five. Evidently this disease rests most heavily upon those whose lives promise the greatest service and who have the greatest earning or wealth-producing power. This fact, however, will automatically carry its due weight into later results. The irregularity in the difference column at the ages two, three, and four, is due to a heavy infant typhoid mortality in one of the states. The

TABLE II.

MORTALITY TABLE WHEN TYPHOID MORTALITY IS NOT PRESENT, WHEN TYPHOID MORTALITY IS PRESENT, AND THE DIFFERENCE IN SURVIVORS AT EACH AGE. AGGREGATE POPULATION, REGISTRATION STATES.

| Age. | Number Living when Typhoid Mortality | | Loss in S'rviv'rs at Each Age. | Age. | Number Living when Typhoid Mortality | | Loss in S'rviv'rs at Each Age. |
|---|---|---|---|---|---|---|---|
| | Is Not Present. | Is Present. | | | Is Not Present. | Is Present. | |
| 0 | 1,302,659 | 1,304,688 | | 50 | 731,303 | 720,505 | 10,798 |
| 1 | 1,107,118 | 1,108,732 | | 51 | 720,376 | 709,553 | 10,823 |
| 2 | 1,060,874 | 1,062,281 | | 52 | 708,993 | 698,161 | 10,832 |
| 3 | 1,042,059 | 1,043,288 | | 53 | 697,115 | 686,291 | 10,824 |
| 4 | 1,030,559 | 1,031,608 | | 54 | 684,711 | 673,914 | 10,797 |
| 5 | 1,022,187 | 1,023,083 | | 55 | 671,762 | 661,009 | 10,753 |
| 6 | 1,016,962 | 1,017,725 | | 56 | 658,257 | 647,566 | 10,691 |
| 7 | 1,012,147 | 1,012,754 | | 57 | 644,179 | 633,569 | 10,610 |
| 8 | 1,007,734 | 1,008,164 | | 58 | 629,512 | 619,001 | 10,511 |
| 9 | 1,003,700 | 1,003,928 | | 59 | 614,238 | 603,846 | 10,392 |
| 10 | 1,000,000 | 1,000,000 | 0 | 60 | 598,343 | 588,089 | 10,254 |
| 11 | 996,574 | 996,321 | 253 | 61 | 581,811 | 571,713 | 10,098 |
| 12 | 993,349 | 992,816 | 533 | 62 | 564,629 | 554,708 | 9,921 |
| 13 | 990,239 | 989,402 | 837 | 63 | 546,791 | 537,064 | 9,727 |
| 14 | 987,147 | 985,984 | 1,163 | 64 | 528,278 | 518,767 | 9,511 |
| 15 | 983,972 | 982,460 | 1,512 | 65 | 509,060 | 499,784 | 9,276 |
| 16 | 980,601 | 978,723 | 1,878 | 66 | 489,104 | 480,085 | 9,019 |
| 17 | 976,923 | 974,662 | 2,261 | 67 | 468,394 | 459,655 | 8,739 |
| 18 | 972,822 | 970,167 | 2,655 | 68 | 446,925 | 438,488 | 8,437 |
| 19 | 968,236 | 965,177 | 3,059 | 69 | 424,729 | 416,618 | 8,111 |
| 20 | 963,182 | 959,714 | 3,468 | 70 | 401,891 | 394,127 | 7,764 |
| 21 | 957,728 | 953,846 | 3,882 | 71 | 378,523 | 371,128 | 7,395 |
| 22 | 951,937 | 947,640 | 4,297 | 72 | 354,745 | 347,738 | 7,007 |
| 23 | 945,879 | 941,166 | 4,713 | 73 | 330,691 | 324,089 | 6,602 |
| 24 | 939,612 | 934,486 | 5,126 | 74 | 306,485 | 300,301 | 6,184 |
| 25 | 933,172 | 927,638 | 5,534 | 75 | 282,232 | 276,480 | 5,752 |
| 26 | 926,580 | 920,647 | 5,933 | 76 | 258,046 | 252,735 | 5,311 |
| 27 | 919,862 | 913,542 | 6,320 | 77 | 234,064 | 229,199 | 4,865 |
| 28 | 913,049 | 906,355 | 6,694 | 78 | 210,437 | 206,021 | 4,416 |
| 29 | 906,159 | 899,107 | 7,052 | 79 | 187,369 | 183,399 | 3,970 |
| 30 | 899,191 | 891,799 | 7,392 | 80 | 165,114 | 161,581 | 3,533 |
| 31 | 892,136 | 884,424 | 7,712 | 81 | 143,923 | 140,814 | 3,109 |
| 32 | 884,982 | 876,970 | 8,012 | 82 | 123,986 | 121,282 | 2,704 |
| 33 | 877,715 | 869,426 | 8,289 | 83 | 105,509 | 103,185 | 2,324 |
| 34 | 870,325 | 861,781 | 8,544 | 84 | 88,588 | 86,617 | 1,971 |
| 35 | 862,811 | 854,032 | 8,779 | 85 | 73,253 | 71,607 | 1,646 |
| 36 | 855,177 | 846,181 | 8,996 | 86 | 59,500 | 58,152 | 1,348 |
| 37 | 847,421 | 838,225 | 9,196 | 87 | 47,426 | 46,343 | 1,083 |
| 38 | 839,539 | 830,156 | 9,383 | 88 | 37,072 | 36,220 | 852 |
| 39 | 831,527 | 821,970 | 9,557 | 89 | 28,326 | 27,672 | 654 |
| 40 | 823,381 | 813,661 | 9,720 | 90 | 21,023 | 20,537 | 486 |
| 41 | 815,096 | 805,223 | 9,873 | 91 | 15,011 | 14,664 | 347 |
| 42 | 806,666 | 796,647 | 10,019 | 92 | 10,216 | 9,980 | 236 |
| 43 | 798,082 | 787,925 | 10,157 | 93 | 6,562 | 6,410 | 152 |
| 44 | 789,325 | 779,039 | 10,286 | 94 | 3,930 | 3,839 | 91 |
| 45 | 780,367 | 769,961 | 10,406 | 95 | 2,159 | 2,109 | 50 |
| 46 | 771,174 | 760,658 | 10,516 | 96 | 1,063 | 1,039 | 24 |
| 47 | 761,708 | 751,096 | 10,612 | 97 | 453 | 442 | 11 |
| 48 | 751,932 | 741,241 | 10,691 | 98 | 157 | 154 | 3 |
| 49 | 741,809 | 731,056 | 10,753 | 99 | 40 | 39 | 1 |
| | | | | 100 | 6 | 5 | 1 |

TABLE III.

COMPARATIVE TABLE SHOWING THE DEATH-RATE PER ANNUM PER 1,000 PERSONS FOR ALL AGES BETWEEN 0 AND 100; WHEN TYPHOID MORTALITY IS PRESENT, WHEN TYPHOID MORTALITY IS NOT PRESENT, AND THE DIFFERENCE. AGGREGATE POPULATION, REGISTRATION STATES.

| Age. | Death-Rate Per Annum Per 1,000 When Typhoid Mortality | | Difference. | Age. | Death-Rate Per Annum Per 1,000 When Typhoid Mortality | | Difference. |
|---|---|---|---|---|---|---|---|
| | Is Present. | Is Not Present. | | | Is Present. | Is Not Present. | |
| 0 | 150.194 | 150.109 | .085 | 50 | 15.200 | 14.940 | .260 |
| 1 | 41.896 | 41.770 | .126 | 51 | 16.055 | 15.801 | .254 |
| 2 | 17.879 | 17.735 | .144 | 52 | 17.002 | 16.753 | .249 |
| 3 | 11.195 | 11.036 | .159 | 53 | 18.035 | 17.793 | .242 |
| 4 | 8.264 | 8.124 | .140 | 54 | 19.149 | 18.912 | .237 |
| 5 | 5.237 | 5.112 | .125 | 55 | 20.337 | 20.104 | .233 |
| 6 | 4.884 | 4.735 | .149 | 56 | 21.615 | 21.387 | .228 |
| 7 | 4.532 | 4.360 | .172 | 57 | 22.994 | 22.769 | .225 |
| 8 | 4.202 | 4.003 | .199 | 58 | 24.483 | 24.263 | .220 |
| 9 | 3.913 | 3.686 | .227 | 59 | 26.094 | 25.878 | .216 |
| 10 | 3.679 | 3.426 | .253 | 60 | 27.846 | 27.630 | .216 |
| 11 | 3.518 | 3.236 | .282 | 61 | 29.744 | 29.532 | .212 |
| 12 | 3.439 | 3.131 | .308 | 62 | 31.808 | 31.592 | .216 |
| 13 | 3.455 | 3.122 | .333 | 63 | 34.069 | 33.858 | .211 |
| 14 | 3.574 | 3.216 | .358 | 64 | 36.593 | 36.379 | .214 |
| 15 | 3.804 | 3.426 | .378 | 65 | 39.415 | 39.202 | .213 |
| 16 | 4.149 | 3.751 | .398 | 66 | 42.555 | 42.343 | .212 |
| 17 | 4.612 | 4.198 | .414 | 67 | 46.050 | 45.835 | .215 |
| 18 | 5.143 | 4.714 | 429 | 68 | 49.876 | 49.664 | .212 |
| 19 | 5.660 | 5.220 | .440 | 69 | 53.985 | 53.771 | .214 |
| 20 | 6.114 | 5.662 | .452 | 70 | 58.354 | 58.145 | .209 |
| 21 | 6.506 | 6.047 | .459 | 71 | 63.024 | 62.818 | .206 |
| 22 | 6.832 | 6.364 | .468 | 72 | 68.008 | 67.806 | .202 |
| 23 | 7.098 | 6.626 | 472 | 73 | 73.400 | 73.198 | .202 |
| 24 | 7.328 | 6.854 | .474 | 74 | 79.324 | 79.133 | .191 |
| 25 | 7.536 | 7.064 | .472 | 75 | 85.883 | 85.695 | .188 |
| 26 | 7.717 | 7.250 | .467 | 76 | 93.125 | 92.937 | .188 |
| 27 | 7.867 | 7.407 | .460 | 77 | 101.126 | 100.942 | .184 |
| 28 | 7.997 | 7.546 | .451 | 78 | 109.804 | 109.620 | .184 |
| 29 | 8.128 | 7.690 | .438 | 79 | 118.965 | 118.776 | .189 |
| 30 | 8.270 | 7.846 | .424 | 80 | 128.524 | 128.342 | .182 |
| 31 | 8.428 | 8.019 | .409 | 81 | 138.708 | 138.525 | .183 |
| 32 | 8.602 | 8.211 | .391 | 82 | 149.214 | 149.025 | .189 |
| 33 | 8.793 | 8.420 | .373 | 83 | 160.566 | 160.375 | .191 |
| 34 | 8.992 | 8.634 | .358 | 84 | 173.292 | 173.105 | .187 |
| 35 | 9.193 | 8.848 | .345 | 85 | 187.901 | 187.747 | .154 |
| 36 | 9.402 | 9.069 | .333 | 86 | 203.071 | 202.924 | .147 |
| 37 | 9.626 | 9.301 | .325 | 87 | 218.436 | 218.319 | .117 |
| 38 | 9.861 | 9.543 | .318 | 88 | 236.002 | 235.919 | .083 |
| 39 | 10.109 | 9.796 | .313 | 89 | 257.842 | 257.820 | .022 |
| 40 | 10.370 | 10.062 | .308 | 90 | 285.972 | ..... | .... |
| 41 | 10.650 | 10.342 | .308 | 91 | 319.422 | ..... | .... |
| 42 | 10.948 | 10.641 | .307 | 92 | 357.715 | ..... | .... |
| 43 | 11.278 | 10.973 | .305 | 93 | 401.092 | ..... | .... |
| 44 | 11.653 | 11.349 | .304 | 94 | 450.638 | ..... | .... |
| 45 | 12.082 | 11.780 | .302 | 95 | 507.349 | ..... | .... |
| 46 | 12.571 | 12.275 | .296 | 96 | 574.591 | ..... | .... |
| 47 | 13.121 | 12.834 | .287 | 97 | 651.584 | ..... | .... |
| 48 | 13.740 | 13.463 | .277 | 98 | 746.753 | ..... | .... |
| 49 | 14.433 | 14.164 | .269 | 99 | 871.795 | ..... | .... |
| | | | | 100 | 1,000.000 | | |

TABLE IV.

COMPARATIVE TABLE SHOWING AGES 0 TO 100, THE TOTAL NUMBER OF YEARS OF FUTURE LIFETIME WHICH WILL BE LIVED BY THE SURVIVORS OF 1,000,000 PERSONS AT AGE 10, AND THE LOSS IN SAME DUE TO THE PRESENCE OF TYPHOID MORTALITY. AGGREGATE POPULATION, REGISTRATION STATES.

| Age. | Total Future Lifetime in Years when Typhoid Mortality | | Loss in Years. | Age. | Total Future Lifetime in Years when Typhoid Mortality | | Loss in Years. |
|---|---|---|---|---|---|---|---|
| | Is Not Present. | Is Present. | | | Is Not Present. | Is Present. | |
| 0 | 60,807,037 | 60,279,181 | 527,856 | 50 | 15,065,007 | 14,795,789 | 269,218 |
| 1 | 59,699,919 | 59,170,449 | 529,470 | 51 | 14,344,631 | 14,086,236 | 258,395 |
| 2 | 58,639,045 | 58,108,168 | 530,877 | 52 | 13,635,638 | 13,388,075 | 247,563 |
| 3 | 57,596,986 | 57,064,880 | 532,106 | 53 | 12,938,523 | 12,701,784 | 236,739 |
| 4 | 56,566,427 | 56,033,272 | 533,155 | 54 | 12,253,812 | 12,027,870 | 225,942 |
| 5 | 55,544,240 | 55,010,189 | 534,051 | 55 | 11,582,050 | 11,366,861 | 215,189 |
| 6 | 54,527,278 | 53,992,464 | 534,814 | 56 | 10,923,793 | 10,719,295 | 204,498 |
| 7 | 53,515,131 | 52,979,710 | 535,421 | 57 | 10,279,614 | 10,085,726 | 193,888 |
| 8 | 52,507,397 | 51,971,546 | 535,851 | 58 | 9,650,102 | 9,466,725 | 183,377 |
| 9 | 51,503,697 | 50,967,618 | 536,079 | 59 | 9,035,864 | 8,862,879 | 172,985 |
| 10 | 50,503,697 | 49,967,618 | 536,079 | 60 | 8,437,521 | 8,274,790 | 162,731 |
| 11 | 49,507,123 | 48,971,297 | 535,826 | 61 | 7,855,710 | 7,703,077 | 152,633 |
| 12 | 48,513,774 | 47,978,481 | 535,293 | 62 | 7,291,081 | 7,148,369 | 142,712 |
| 13 | 47,523,535 | 46,989,079 | 534,456 | 63 | 6,744,290 | 6,611,305 | 132,985 |
| 14 | 46,536,388 | 46,003,095 | 533,293 | 64 | 6,216,012 | 6,092,538 | 123,474 |
| 15 | 45,552,416 | 45,020,635 | 531,781 | 65 | 5,706,952 | 5,592,754 | 114,198 |
| 16 | 44,571,815 | 44,041,912 | 529,903 | 66 | 5,217,848 | 5,112,669 | 105,179 |
| 17 | 43,594,892 | 43,067,250 | 527,642 | 67 | 4,749,454 | 4,653,014 | 96,440 |
| 18 | 42,622,070 | 42,097,083 | 524,987 | 68 | 4,302,529 | 4,214,526 | 88,003 |
| 19 | 41,653,834 | 41,131,906 | 521,928 | 69 | 3,877,800 | 3,797,908 | 79,892 |
| 20 | 40,690,652 | 40,172,192 | 518,460 | 70 | 3,475,909 | 3,403,781 | 72,128 |
| 21 | 39,732,924 | 39,218,346 | 514,578 | 71 | 3,097,386 | 3,032,653 | 64,733 |
| 22 | 38,780,987 | 38,270,706 | 510,281 | 72 | 2,742,641 | 2,684,915 | 57,726 |
| 23 | 37,835,108 | 37,329,540 | 505,568 | 73 | 2,411,950 | 2,360,826 | 51,124 |
| 24 | 36,895,496 | 36,395,054 | 500,442 | 74 | 2,105,465 | 2,060,525 | 44,940 |
| 25 | 35,962,324 | 35,467,416 | 494,908 | 75 | 1,823,233 | 1,784,045 | 39,188 |
| 26 | 35,035,744 | 34,546,769 | 488,975 | 76 | 1,565,187 | 1,531,310 | 33,877 |
| 27 | 34,115,882 | 33,633,227 | 482,655 | 77 | 1,331,123 | 1,302,111 | 29,012 |
| 28 | 33,202,833 | 32,726,872 | 475,961 | 78 | 1,120,686 | 1,096,090 | 24,596 |
| 29 | 32,296,674 | 31,827,765 | 468,909 | 79 | 933,317 | 912,691 | 20,626 |
| 30 | 31,397,483 | 30,935,966 | 461,517 | 80 | 768,203 | 751,110 | 17,093 |
| 31 | 30,505,347 | 30,051,542 | 453,805 | 81 | 624,280 | 610,296 | 13,984 |
| 32 | 29,620,365 | 29,174,572 | 445,793 | 82 | 500,294 | 489,014 | 11,280 |
| 33 | 28,742,650 | 28,305,146 | 437,504 | 83 | 394,785 | 385,829 | 8,956 |
| 34 | 27,872,325 | 27,443,365 | 428,960 | 84 | 306,197 | 299,212 | 6,985 |
| 35 | 27,009,514 | 26,589,333 | 420,181 | 85 | 232,944 | 227,605 | 5,339 |
| 36 | 26,154,337 | 25,743,152 | 411,185 | 86 | 173,444 | 169,453 | 3,991 |
| 37 | 25,306,916 | 24,904,927 | 401,989 | 87 | 126,018 | 123,110 | 2,908 |
| 38 | 24,467,377 | 24,074,771 | 392,606 | 88 | 88,946 | 86,890 | 2,056 |
| 39 | 23,635,850 | 23,252,801 | 383,049 | 89 | 60,620 | 59,218 | 1,402 |
| 40 | 22,812,469 | 22,439,140 | 373,329 | 90 | 39,597 | 38,681 | 916 |
| 41 | 21,997,373 | 21,633,917 | 363,456 | 91 | 24,586 | 24,017 | 569 |
| 42 | 21,190,707 | 20,837,270 | 353,437 | 92 | 14,370 | 14,037 | 333 |
| 43 | 20,392,625 | 20,049,345 | 343,280 | 93 | 7,808 | 7,627 | 181 |
| 44 | 19,603,300 | 19,270,306 | 332,994 | 94 | 3,878 | 3,788 | 90 |
| 45 | 18,822,933 | 18,500,345 | 322,588 | 95 | 1,719 | 1,679 | 40 |
| 46 | 18,051,759 | 17,739,687 | 312,072 | 96 | 656 | 640 | 16 |
| 47 | 17,290,051 | 16,988,591 | 301,460 | 97 | 203 | 198 | 5 |
| 48 | 16,538,119 | 16,247,350 | 290,769 | 98 | 46 | 44 | 2 |
| 49 | 15,796,310 | 15,516,294 | 280,016 | 99 | 6 | 5 | 1 |
| | | | | 100 | | | |

TABLE V.

COMPARATIVE TABLE SHOWING AT AGES 0 TO 100 THE COMPLETE EXPECTATION OF LIFE WHEN TYPHOID MORTALITY IS NOT PRESENT, WHEN PRESENT, AND THE LOSS IN DAYS DUE TO THE PRESENCE OF TYPHOID MORTALITY. AGGREGATE POPULATION, REGISTRATION STATES.

| Age. | Complete Expectation of Life when Typhoid Mortality | | Loss in Days. | Age. | Complete Expectation of Life when Typhoid Mortality | | Loss in Days. |
|---|---|---|---|---|---|---|---|
| | Is Not Present. | Is Present. | | | Is Not Present. | Is Present. | |
| 0 | 47.179 | 46.702 | 174 | 50 | 21.100 | 21.035 | 24 |
| 1 | 54.424 | 53.868 | 203 | 51 | 20.413 | 20.352 | 22 |
| 2 | 55.774 | 55.201 | 209 | 52 | 19.732 | 19.676 | 20 |
| 3 | 55.772 | 55.197 | 210 | 53 | 19.060 | 19.008 | 19 |
| 4 | 55.389 | 54.816 | 209 | 54 | 18.396 | 18.348 | 18 |
| 5 | 54.839 | 54.269 | 208 | 55 | 17.741 | 17.696 | 16 |
| 6 | 54.118 | 53.552 | 207 | 56 | 17.095 | 17.053 | 15 |
| 7 | 53.373 | 52.813 | 204 | 57 | 16.458 | 16.419 | 14 |
| 8 | 52.604 | 52.051 | 202 | 58 | 15.829 | 15.794 | 13 |
| 9 | 51.814 | 51.268 | 199 | 59 | 15.210 | 15.177 | 12 |
| 10 | 51.004 | 50.468 | 196 | 60 | 14.601 | 14.571 | 11 |
| 11 | 50.177 | 49.652 | 192 | 61 | 14.002 | 13.974 | 10 |
| 12 | 49.339 | 48.825 | 188 | 62 | 13.413 | 13.387 | 9 |
| 13 | 48.492 | 47.992 | 183 | 63 | 12.834 | 12.810 | 9 |
| 14 | 47.642 | 47.157 | 177 | 64 | 12.267 | 12.244 | 8 |
| 15 | 46.794 | 46.324 | 172 | 65 | 11.711 | 11.690 | 8 |
| 16 | 45.954 | 45.499 | 166 | 66 | 11.168 | 11.150 | 7 |
| 17 | 45.125 | 44.687 | 160 | 67 | 10.640 | 10.623 | 6 |
| 18 | 44.313 | 43.892 | 154 | 68 | 10.127 | 10.111 | 6 |
| 19 | 43.520 | 43.116 | 147 | 69 | 9.630 | 9.616 | 5 |
| 20 | 42.746 | 42.359 | 141 | 70 | 9.149 | 9.136 | 5 |
| 21 | 41.987 | 41.616 | 135 | 71 | 8.683 | 8.671 | 4 |
| 22 | 41.239 | 40.885 | 129 | 72 | 8.231 | 8.221 | 4 |
| 23 | 40.500 | 40.163 | 123 | 73 | 7.794 | 7.784 | 4 |
| 24 | 39.767 | 39.447 | 117 | 74 | 7.370 | 7.362 | 3 |
| 25 | 39.038 | 38.734 | 111 | 75 | 6.960 | 6.953 | 3 |
| 26 | 38.312 | 38.024 | 105 | 76 | 6.566 | 6.559 | 3 |
| 27 | 37.588 | 37.316 | 99 | 77 | 6.187 | 6.181 | 2 |
| 28 | 36.865 | 36.608 | 94 | 78 | 5.826 | 5.820 | 2 |
| 29 | 36.141 | 35.903 | 87 | 79 | 5.481 | 5.477 | 1 |
| 30 | 35.417 | 35.189 | 83 | 80 | 5.153 | 5.149 | 1 |
| 31 | 34.694 | 34.479 | 78 | 81 | 4.838 | 4.834 | 1 |
| 32 | 33.970 | 33.767 | 74 | 82 | 4.535 | 4.532 | 1 |
| 33 | 33.247 | 33.056 | 70 | 83 | 4.242 | 4.239 | 1 |
| 34 | 32.525 | 32.345 | 66 | 84 | 3.956 | 3.954 | 1 |
| 35 | 31.804 | 31.634 | 62 | 85 | 3.680 | 3.679 | 0 |
| 36 | 31.084 | 30.923 | 59 | 86 | 3.415 | 3.414 | |
| 37 | 30.363 | 30.212 | 55 | 87 | 3.157 | 3.156 | |
| 38 | 29.644 | 29.500 | 53 | 88 | 2.899 | 2.899 | |
| 39 | 28.925 | 28.789 | 50 | 89 | 2.640 | 2.640 | |
| 40 | 28.206 | 28.078 | 47 | 90 | 2.384 | 2.383 | |
| 41 | 27.487 | 27.367 | 44 | 91 | 2.138 | 2.138 | |
| 42 | 26.769 | 26.656 | 41 | 92 | 1.907 | 1.907 | |
| 43 | 26.052 | 25.946 | 39 | 93 | 1.690 | 1.690 | |
| 44 | 25.336 | 25.236 | 37 | 94 | 1.487 | 1.487 | |
| 45 | 24.621 | 24.528 | 34 | 95 | 1.296 | 1.296 | |
| 46 | 23.908 | 23.822 | 31 | 96 | 1.118 | 1.116 | |
| 47 | 23.199 | 23.118 | 30 | 97 | .948 | .948 | |
| 48 | 22.494 | 22.419 | 27 | 98 | .793 | .786 | |
| 49 | 21.794 | 21.724 | 26 | 99 | .650 | .628 | |

effect of typhoid fever on the community is made more apparent by an inspection of Table IV.

The third column of this table gives at each age for a community with typhoid fever present the total number of years of future lifetime remaining to the survivors of an original group of 1,000,000 persons at age ten. It is obtained by summing the numbers given in the third column of Table II from the bottom up to the age required. The second column, obtained from the second column of Table II, gives similar results for a community in which there are no deaths from typhoid fever, while the difference, given in the fourth column, shows the number of years of lifetime which would be gained were there no deaths from typhoid. For example, the group of persons who have attained the age of twenty years have 40,172,192 years yet to live. If typhoid were not present the years of future lifetime would be 40,690,652. Thus they have sustained a loss of 518,460 years from a preventable disease.

It should be remembered that the heaviest loss is at the younger ages and the elimination of typhoid mortality does not mean so much a lengthening of the period of old age, as a broadening of the active or wealth-producing period. This is revealed by the fact that of the 518,460 years of future lifetime, which are lost by the group who attain to the age of twenty years, 355,729 years are lost between ages twenty and sixty.

Possibly it is of greater interest to know the average future lifetime of the individuals of this group under the two conditions—typhoid present as now, and typhoid not present—together with the difference. This is exhibited in Table V, where it is seen that at the younger ages, the loss of future lifetime averages as much as half a year for each individual of the group. This average future lifetime is known as the complete expectation of life, and is obtained from the preceding table by dividing the total future lifetime at any age by the number living at that age, and adding to the quotient one half year. This one half year is the average length of life in the year in which death occurs, on the assumption that the deaths occur uniformly throughout the year.

We shall next consider the economic loss to the community,

brought about by this disease. Before proceeding to tables setting forth such values, let us notice the figures in Table IV. Between the ages twenty and seventy, the total number of years lost by the group of age twenty is approximately 450,000. If each one in this group during the years of his age from twenty to seventy has an annual wealth-producing capacity of $100, then the community sustains through this one group a loss of $45,000,000 due to typhoid mortality. This assumes nearly 960,000 people in the group at age twenty. According to the census of 1900 there were over 740,000 young men at age twenty in the United States. On a group of this size the loss is about $35,000,000. This loss is spread over fifty years and its present value is much less, but the illustration shows wherein the monetary loss to the community lies. The exact amounts of loss appear in the tables which follow.

On the basis of the mortality tables exhibited in Table II, corresponding sets of annuity tables were computed which appear in Table VI.

Table VI gives at the various ages up to sixty-nine, the value, capitalized at five per cent., of a life annuity of $100 per annum, payable at the end of each year until age seventy; stated in other words, the amount which, improved at five per cent. interest, would pay to the one at the given age, $100 each year of life up to the age of seventy. These values are found by well-known methods which need not be discussed here. To illustrate, if there were deposited in trust at five per cent. interest, for every person in this group whose age is twenty, $1,578.50, and if at the end of each year $100 were paid to all survivors, the fund would suffice to make the payments until age seventy and would then be exhausted. In the second column of Table VI we find that $1,587.71 is the amount required to make similar payments in a group where there are no deaths from typhoid fever. Or as appears in the fourth column, $9.21 more is required for each individual, since under these better conditions, people live longer, thus receiving additional payments from the fund. It is equally true that if the individual has a wealth-producing capacity of $100 per annum, the efforts of those at age twenty will yield to the community an average of $1,578.50 each, and

TABLE VI.

PRESENT OR CAPITALIZED VALUE AT EACH AGE FROM 0 TO 70 COMPUTED WITH 5 PER CENT. INTEREST, OF A WEALTH INCREMENT OR SAVING OF $100 PER ANNUM AT THE END OF EACH YEAR UNTIL AGE 70. BASED ON AGGREGATE POPULATION, REGISTRATION STATES.

| Age. | When Typhoid Mortality | | Loss in Value at Each Age. | Age. | When Typhoid Mortality | | Loss in Value at Each Age. |
|---|---|---|---|---|---|---|---|
| | Is Not Present. | Is Present. | | | Is Not Present. | Is Present. | |
| 0 | $1,413.19 | $1,406.79 | $6.40 | 35 | $1,388.33 | $1,383.43 | $4.90 |
| 1 | 1,645.93 | 1,638.19 | 7.74 | 36 | 1,370.76 | 1,366.08 | 4.68 |
| 2 | 1,703.56 | 1,695.32 | 8.24 | 37 | 1,352.48 | 1,348.00 | 4.48 |
| 3 | 1,721.03 | 1,712.49 | 8.54 | 38 | 1,333.43 | 1,329.16 | 4.27 |
| 4 | 1,727.25 | 1,718.47 | 8.78 | 39 | 1,313.59 | 1,309.51 | 4.08 |
| 5 | 1,728.47 | 1,719.43 | 9.04 | 40 | 1,292.92 | 1,289.03 | 3.89 |
| 6 | 1,724.21 | 1,714.91 | 9.30 | 41 | 1,271.36 | 1,267.66 | 3.70 |
| 7 | 1,719.04 | 1,709.49 | 9.55 | 42 | 1,248.88 | 1,245.37 | 3.51 |
| 8 | 1,712.89 | 1,703.14 | 9.75 | 43 | 1,225.43 | 1,222.12 | 3.31 |
| 9 | 1,705.77 | 1,695.84 | 9.93 | 44 | 1,200.98 | 1,197.86 | 3.12 |
| 10 | 1,697.68 | 1,687.63 | 10.05 | 45 | 1,175.50 | 1,172.58 | 2.92 |
| 11 | 1,688.70 | 1,678.55 | 10.15 | 46 | 1,148.99 | 1.146.27 | 2.72 |
| 12 | 1,678.89 | 1,668.70 | 10.19 | 47 | 1,121.43 | 1,118.91 | 2.52 |
| 13 | 1,668.37 | 1,658.18 | 10.19 | 48 | 1,092.81 | 1,090.47 | 2.34 |
| 14 | 1,657.27 | 1,647.13 | 10.14 | 49 | 1,063.11 | 1,060.95 | 2.16 |
| 15 | 1,645.75 | 1,635.69 | 10.06 | 50 | 1,032.31 | 1,030.31 | 2.00 |
| 16 | 1,633.98 | 1,624.03 | 9.95 | 51 | 1,000.36 | 998.52 | 1.84 |
| 17 | 1,622.14 | 1,612.34 | 9.80 | 52 | 967.24 | 965.56 | 1.68 |
| 18 | 1,610.42 | 1,600.80 | 9.62 | 53 | 932.91 | 931.37 | 1.54 |
| 19 | 1,598.95 | 1,589.53 | 9.42 | 54 | 897.30 | 895.90 | 1.40 |
| 20 | 1,587.71 | 1,578.50 | 9.21 | 55 | 860.33 | 859.06 | 1.27 |
| 21 | 1,576.59 | 1,567.63 | 8.96 | 56 | 821.88 | 820.74 | 1.14 |
| 22 | 1,565.49 | 1,556.79 | 8.70 | 57 | 781.83 | 780.81 | 1.02 |
| 23 | 1,554.29 | 1,545.87 | 8.42 | 58 | 740.05 | 739.15 | .90 |
| 24 | 1,542.89 | 1.534.77 | 8.12 | 59 | 696.37 | 695.58 | .79 |
| 25 | 1,531.22 | 1,523.40 | 7.82 | 60 | 650.62 | 649.93 | .69 |
| 26 | 1,519.22 | 1,511.72 | 7.50 | 61 | 602.56 | 601.97 | .59 |
| 27 | 1,506.83 | 1,499.65 | 7.18 | 62 | 551.94 | 551.45 | .49 |
| 28 | 1,493.98 | 1,487.12 | 6.86 | 63 | 498.44 | 498.04 | .40 |
| 29 | 1,480.60 | 1,474.06 | 6.54 | 64 | 441.71 | 441.39 | .32 |
| 30 | 1,466.68 | 1.460.45 | 6.23 | 65 | 381.30 | 381.06 | .24 |
| 31 | 1,452.19 | 1,446.26 | 5.93 | 66 | 316.70 | 316.53 | .17 |
| 32 | 1,437.13 | 1,431.48 | 5.65 | 67 | 247.24 | 247.13 | .11 |
| 33 | 1,421.48 | 1.416.09 | 5.39 | 68 | 172.07 | 172.02 | 05 |
| 34 | 1,405.22 | 1,400.09 | 5.13 | 69 | 90.12 | 90.10 | .02 |

that this amount would be increased to $1,587.71 if the mortality from typhoid were eliminated. Thus in this one group is shown a loss caused by deaths from typhoid fever which averages $9.21 for every member of the group.

To find for any community the monetary loss due to this disease, the male population at each age is required. This number multiplied by the loss for that age as shown in the fourth column of Table VI, gives the loss on the basis of a

wealth-producing capacity of $100 per annum from the given age to age seventy. If the wealth-producing capacity is greater, the loss is greater in the same ratio. Such an application is made in Table VII for the male population of the United States in 1900.

TABLE VII.

TABLE SHOWING THE CAPITALIZED OR PRESENT VALUE, COMPOUNDED ANNUALLY, AT 5 PER CENT. OF THE LOSS DUE TO TYPHOID FEVER ON THE MALE POPULATION OF THE UNITED STATES FOR EACH AGE AND CERTAIN AGE GROUPS BETWEEN THE AGES 20 AND 60 ON THE BASIS OF A WEALTH-PRODUCING CAPACITY OF $100 PER ANNUM UNTIL AGE 70. BASED ON AGGREGATE POPULATION, REGISTRATION STATES.

| Age. | Population, 1900. | Loss Rate. | Total Loss. | Age. | Population, 1900. | Loss Rate. | Total Loss. |
|---|---|---|---|---|---|---|---|
| 20 | 743,687 | 9.21 | $6,849,357.27 | 40 | 643,551 | 3.89 | $ 2,503,413.39 |
| 21 | 739,047 | 8.96 | 6,621,861.12 | 41 | 393,935 | 3.70 | 1,457,559.50 |
| 22 | 745,491 | 8.70 | 6,485,771.70 | 42 | 451,803 | 3.51 | 1,585,828.53 |
| 23 | 721,847 | 8.42 | 6,077,951.74 | 43 | 390,257 | 3.31 | 1,291,750.67 |
| 24 | 734,301 | 8.12 | 5,962,524.12 | 44 | 389,226 | 3.12 | 1,214,385.12 |
| 20–24 | 3,684,373 | | 31,997,465.95 | 20–44 | 14,799,693 | | 92,911,492.23 |
| 25 | 745,136 | 7.82 | 5,826,963.52 | 45 | 476,468 | 2.92 | 1,391,286.56 |
| 26 | 673,529 | 7.50 | 5,051,467.50 | 46 | 349,745 | 2.72 | 951,306.40 |
| 27 | 666,166 | 7.18 | 4,783,071.88 | 47 | 337,687 | 2.52 | 850,971.24 |
| 28 | 684,941 | 6.86 | 4,698,695.26 | 48 | 349,707 | 2.34 | 818,314.38 |
| 29 | 599,305 | 6.54 | 3,919,454.70 | 49 | 331,628 | 2.16 | 716,316.48 |
| 20–29 | 6,963,450 | | 56,277,118.81 | 20–49 | 16,644,928 | | 97,639,687.29 |
| 30 | 777,266 | 6.23 | 4,842,367.18 | 50 | 454,433 | 2.00 | 908,866.00 |
| 31 | 502,398 | 5.93 | 2,979,220.14 | 51 | 276,208 | 1.84 | 508,222.72 |
| 32 | 579,386 | 5.65 | 3,273,530.90 | 52 | 310,129 | 1.68 | 521,016.72 |
| 33 | 543,950 | 5.39 | 2,931,890.50 | 53 | 264,837 | 1.54 | 407,848.98 |
| 34 | 528,037 | 5.13 | 2,708,829.81 | 54 | 263,666 | 1.40 | 369,132.40 |
| 20–34 | 9,894,487 | | 73,012,957.34 | 20–54 | 18,214,201 | | 100,354,774.11 |
| 35 | 608,152 | 4.90 | 2,979,944.80 | 55 | 297,606 | 1.27 | 377,959.62 |
| 36 | 492,186 | 4.68 | 2,303,430.48 | 56 | 244,362 | 1.14 | 278,572.68 |
| 37 | 475,287 | 4.48 | 2,129,285.76 | 57 | 207,971 | 1.02 | 212,130.42 |
| 38 | 551,768 | 4.27 | 2,356,049.36 | 58 | 202,677 | .90 | 182,409.30 |
| 39 | 509,041 | 4.08 | 2,076,887.28 | 59 | 195,194 | .79 | 154,203.26 |
| | | | | 60 | 275,887 | .69 | 190,362.03 |
| 20–39 | 12,530,921 | | 84,858,555.02 | 20–60 | 19,637,898 | | 101,750,411.42 |

The total loss, $101,750,411, seems very great, but it must be remembered that even in obtaining this figure, no account is taken of males younger than twenty or older than sixty, or of females; that in all these classes are large numbers of wealth-producing individuals; and further that the average wealth-producing capacity of those considered is several times $100 per annum. We also repeat that these computa-

tions are based upon the reports from registration states, where in many sections the mortality from typhoid has already been reduced to near the minimum residual, very greatly decreasing the average loss for the whole region.

The final sum, $101,750,411, which is the present value of the total loss may be reduced to annual loss by determining the annual payment on a fifty-year annuity whose present value at five per cent. is equal to the total loss. The resulting annual loss is $5,573,555. At the end of fifty years those now twenty will have attained to the age of seventy and an entirely new group will have appeared, hence the choice of fifty years as the basis for the computation of annual loss. The annual loss here determined is derived from the total loss sustained by a specified group, and no account is taken of persons below age twenty. We note, however, that in one year the whole group of those aged nineteen will have entered the age period with which we are dealing, and each succeeding year other groups enter, whose losses during the fifty years will swell the annual loss.

In Table VIII is exhibited the total and annual loss for the United States, based upon the population in 1900, also upon the population in 1910, as reported by the Census Bureau.

TABLE VIII.

MINIMUM ANNUAL AND TOTAL LOSSES FROM TYPHOID FEVER IN THE UNITED STATES CAPITALIZED AT 5 PER CENT.

| Year. | *Population. | Capitalized Total Loss. | Capitalized Annual Loss. |
|---|---|---|---|
| 1900 | 76,212,168 | $101,750,411.42 | $5,573,555.37 |
| 1910 | 92,228,531 | 123,133,762.24 | 6,744,865.51 |

* Continental United States, Alaska and Hawaii, U. S. Census.

It seems desirable to combine the results given in this paper into a formula for estimating the capitalized value of the removable loss from impure water for a community whose population and typhoid death-rate per 100,000 are known. The following formula is based upon the assumption already made, that the removable loss in a community due to impure water is equal to the total loss

due to typhoid mortality in the fixed group heretofore considered; and upon the further assumption that the total community loss due to impure water is proportional to the removable typhoid death-rate per 100,000. These assumptions while only approximately correct, may be considered, because of the conservative character of the original results, to give minimum losses for the area in question.

The formula for total loss is:

$$\text{Loss} = \frac{(t-r)}{31.3-15} \cdot \frac{101750411.42}{76212168} \cdot p.$$

In this formula $t$ is the typhoid death-rate per 100,000 for the community, $r$ is the estimated residual typhoid death-rate per 100,000, and $p$ is the total population.

This formula simplifies to:

$$\text{Loss} = .0819\,(t-r)\,p.$$

The annual loss may be obtained by dividing the total loss by 18.256.

Table IX gives the total and annual community losses for certain cities as estimated by means of this formula, the typhoid residual $r$ being assumed to be 15 per cent.

TABLE IX.

ESTIMATED TOTAL AND ANNUAL LOSSES DUE TO IMPURE WATER IN CERTAIN CITIES OF THE UNITED STATES CAPITALIZED AT 5 PER CENT.

| City. | Population in 1910. | *Death Rate per 100,000 | Estimated Present Value of the Total Loss. | Estimated Annual Loss. |
|---|---|---|---|---|
| Chicago, Ill. | 2,185,283 | 25.3 | $1,843,439.18 | $100,977.58 |
| Cincinnati, Ohio. | 363,591 | 50.7 | 1,063,078.27 | 58,231.96 |
| Detroit, Mich. | 465,766 | 20.9 | 225,062.79 | 12,328.20 |
| Grand Rapids, Mich. | 112,571 | 40.4 | 234,176.95 | 12,827.45 |
| Indianapolis, Ind. | 233,650 | 39.3 | 465,003.22 | 25,471.36 |
| Minneapolis, Minn. | 301,408 | 34.9 | 491,237.77 | 26,908.40 |
| Philadelphia, Pa. | 1,549,008 | 53.6 | 4,896,940.95 | 268,238.44 |
| Pittsburg, Pa. | 533,905 | 116.9 | 4,455,762.91 | 244,072.15 |

*Engineering News, Vol. 63, No. 16, April 21, 1910, p. 453.

In calculating the total and annual capitalized losses the populations are those taken from the official reports of the last census, 1910, and the typhoid death-rate assumed is that given in *Engineering News* as the average rate for the period of years 1898–1908.

Finally we wish to call attention again to the fact that a wealth-producing capacity of $100 per annum for each individual is taken simply as a unit upon which to base the tables, that no assumption is made concerning the *actual* average wealth-producing capacity. The contention of this paper is that the results here given are conservative estimates upon the unit chosen. To find the actual losses these results should be multiplied by the average annual wealth-producing capacity per individual measured in units of $100. The determination of the latter quantity is an economic problem not considered here.

## II.

The remainder of this paper is a description of the processes by which the life tables are computed. The formulæ used, with the exception of those dealing with the osculatory differences, for the derivation of which the reader is referred to King,* are well known formulæ of interpolation and no reference to their source of derivation need be given.

The figures in Table I are taken directly from the reports of the census, except that the number of deaths due to typhoid fever is given in those reports only in age groups of ten years each. To obtain the number at intervals of five years the following central difference formula of interpolation is used:

$$u_{0.5}=\frac{1}{2}(u_0+u_1)-\frac{1}{16}\delta^2(u_0+u_1)+\frac{3}{256}\delta^4(u_0+u_1).\dagger \qquad (1)$$

This formula enables us to make the interpolation for ages 25, 35.....75. To get the number at age eighty-five the ordinary difference formula is used in the form

$$u_{3.5}=u_0+3.5\,\Delta u_0+4.375\,\Delta^2 u_0+2.1875\,\Delta^3 u_0+ \qquad (2)$$
$$0.2734375\,\Delta^4 u_0$$

* *Loc. cit.*, p. 242.

† Bessel's formula: Given in a paper by James Buchanan: "Osculatory Interpolation by Central Differences, with an Application to Life Table Construction," *Journal of the Institute of Actuaries*, Vol. 42, p. 394.

with $u_0$ corresponding to age fifty. Again the same formula in the form

$$u_{2.5} = u_0 + 2.5\,\Delta u_0 + 1.875\,\Delta^2 u_0 + 0.3125\,\Delta^3 u_0 - 0.0390625\,\Delta^4 u_0, \qquad (3)$$

is used to obtain the number at age fifteen by reversing the table of differences and letting $u_0$ correspond to age forty. The number at age five is given in the census report.

The two mortality tables are constructed by the same methods, one based upon the numbers in the second and third columns of Table I, and the other upon the numbers in the second and fifth columns. They represent respectively the actual mortality and the mortality upon the supposition that no deaths occur due to typhoid fever. The following are the steps in the construction.

The sum of the numbers in the second column of Table I from the end of the column up to any given age is the total number living at that age and over, and is denoted by the symbol $T_x$. Thus $T_{35}$ is 6,864,236. The number living at the given age $x$ is found by subtracting the number living at age $x+1$ and over from the number living at age $x$ and over. This is indicated by the symbol $L_x$, and we have $L_x = T_x - T_{x+1}$. We have $T_x$ at the various five-year intervals $T_5, T_{10} \ldots\ldots T_{90}$. To get $L_x$ at these ages we need $T_6$, $T_{11} \ldots\ldots T_{91}$.

These are obtained by interpolation from the known values of $T$ by employing the following central difference formula:

$$u_x = u_{-2} + (2+x)\,\Delta u_{-2} + \frac{(2+x)(1+x)}{2!}\,\Delta^2 u_{-2} + \frac{(2+x)(1+x)x}{3!}\,\Delta^3 u_{-2} + \frac{(2+x)(1+x)x(x-1)}{4!}\,\Delta^4 u_{-2}. \qquad (4)$$

Let $x = 0.2$ and let $u_0$ be successively $T_{15}, T_{20} \ldots\ldots T_{90}$, thus getting values of $T_{16}, T_{21} \ldots\ldots T_{91}$. By using the same formula adapted to two differences in the forms

$$u_x = u_{-1} + (1+x)\,\Delta u_{-1} + \frac{(1+x)x}{2!}\,\Delta^2 u_{-1} \qquad (5)$$

and

$$u_x = u_0 + x\,\Delta u_0 + \frac{x(x-1)}{2!}\,\Delta^2 u_0, \qquad (6)$$

we get $T_{11}$ and $T_6$ thus:

$$T_{11} = T_5 + 1.2\,\Delta T_5 + .12\,\Delta^2 T_5, \tag{7}$$

$$T_6 = T_5 + .2\,\Delta T_5 - .08\,\Delta^2 T_5. \tag{8}$$

Then by subtraction we have directly the values of $L_x$ for $x = 5, 10 \ldots\ldots 90$.

If we sum the third column of Table I from the bottom upward, we obtain for each age a quantity $l_x$ which corresponds to the $T_x$ obtained from the second column. The same methods and formulæ used above in getting $T_{x+1}$ and $L_x$ are employed here to find $l_{x+1}$ and a new column $d_x$. The number $l_x$ is the sum of all those who die at age $x$ and above, hence $l_x - l_{x+1}$ is the number who die in the year of life $x$ and is denoted by the symbol $d_x$. Having for age $x$ the number living $l_x$, and the number of deaths $d_x$, the probability of death in the year of life $x$, denoted by $q_x$, follows at once by applying the formula:

$$q_x = \frac{d_x}{L_x + 0.5 d_x}. \tag{9}$$

We have now a column $q_x$ which represents the original data at the ages 5, 10, 15 ..... 85. The osculatory method is then employed to complete this table, that is, to find the values of $q_x$ at the intervening ages. Let $\Delta, \Delta^2 \ldots\ldots \Delta^5$ denote the quinquennial differences of the column $q_x$ and let $\delta, \delta^2 \ldots\ldots \delta^5$ denote the subdivided osculatory differences of $q_x$. The formulæ connecting quinquennial and subdivided osculatory differences are:

(10)

$$\delta\, u_{10} = .2\,\Delta u_0 + .32\,\Delta^2 u_0 + .088\,\Delta^3 u_0 - .0176\,\Delta^4 u_0 + .0016\,\Delta^5 u_0$$

$$\delta^2 u_{10} = .04\,\Delta^2 u_0 + .048\,\Delta^3 u_0 + .0016\,\Delta^4 u_0 + .0048\,\Delta^5 u_0$$

$$\delta^3 u_{10} = .008\,\Delta^3 u_0 + .0064\,\Delta^4 u_0 - .0048\,\Delta^5 u^0$$

$$\delta^4 u_{10} = .0016\,\Delta^4 u_0 - .0032\,\Delta^5 u_0$$

$$\delta^5 u_{10} = .008\,\Delta^5 u_0$$

These yield values of $\delta$, $\delta^2$, $\delta^3$, $\delta^4$, $\delta^5$ at each of the ages 15, 20, 25 ..... 60 and the values of $q_x$ at every age from fifteen to seventy-five follow at once. In deriving values

of $q_x$ between the ages five and fifteen we reverse the table of $\delta$'s and using $q_{15}$, $q_{16}$, $q_{17}$, we find $\delta q_{17}$ and $\delta^2 q_{17}$, then

$$\delta^3 q_{17} = \tfrac{1}{1\,9}\tfrac{1}{2\,5}(99q_{10} - 7q_5 - 92q_{17} - 609\delta q_{17}\delta - 1617\delta^2 q_{17}), \quad (11)$$

$$\delta^4 q_{17} = \tfrac{1}{3\,5}(q_{10} - q_{17} - 7\delta q_{17} - 21\delta^2 q_{17} - 35\delta^3 q_{17}). \quad (12)$$

Using $\delta^4 q_{17}$ as constant, $q_{14}$, $q_{13}$, etc., are obtained, reproducing correctly $q_{10}$ and $q_5$. The census reports give numbers at each year of age for ages younger than five, so that $q_0$, $q_1$, $q_2$, $q_3$ and $q_4$ are at hand without interpolation.

It should be noted here that the values of $q_x$ at ages younger than seventy-five represent the original data accurately at ages that are multiples of five, and that in forming the curve having as one axis, age, and for the other, probability of dying, the various interpolation curves are so joined that they have a common slope and radius of curvature at these points of junction.

The annuity values from which the financial loss due to typhoid fever is derived, being dependent only upon the value of $q_x$ at ages younger than seventy, cannot be vitiated by what may appear to be arbitrary methods of graduation used at the higher ages. The data obtained from the census concerning ages of death beyond ninety were only sufficient to give us $d_{85}$. To obtain $d_{90}$, $l_{91}$ is computed by applying the ordinary interpolation formula to the $l_x$ column in the form

$$l_{91} = l_{85} + 1.2\,\Delta l_{85} + .12\,\Delta^2 l_{85}. \quad (13)$$

Having $l_{91}$, $d_{90}$ and $q_{90}$ follow directly. At this point the attempt was made to bring the table to an end by assuming $q_{105}$ to be unity. After some experimenting and study of the data, it was decided to bring the table to an end at age 100. Consequently $q_{100}$ is given the value unity. We have $q_{90}$ and therefore $\Delta^3 q_{75}$. We use the ordinary fifth difference interpolation formula, assume the fifth difference to be zero, $q_{100}$ to be unity, and solve for $\Delta^4 q_{75}$.

$$\Delta^4 q_{75} = \tfrac{1}{5}(q_{100} - q_{75} - 5\,\Delta q_{75} - 10\,\Delta^2 q_{75} - 10\,\Delta^3 q_{75}). \quad (14)$$

Using this fourth difference, we derive at once a value of

$q_{95}$. The corresponding subdivided osculatory differences are computed and $q_{76}$, $q_{77}.....q_{80}$ are produced. To get $q_{81}$, $q_{82}.....q_{90}$ the formulæ for subdivided osculatory third differences apply. They are:

$$\begin{aligned} \delta\, u_5 &= .2\,\Delta u_0 + .12\,\Delta^2 u_0 - .016\,\Delta^3 u_0 \\ \delta^2 u_5 &= .04\,\Delta^2 u_0 - .016\,\Delta^3 u_0 \\ \delta^3 u_5 &= .024\,\Delta^3 u_0 \end{aligned} \qquad (15)$$

The values of $q_x$ from age ninety to age one hundred are obtained by the same method as that employed at the ages between five and fifteen, finding $\delta^3 q_{88}$ and $\delta^4 q_{88}$. The table $q_x$ having been completed, the life-table $l_x$ is built upon a radix of one million at age ten, the equation $l_{x+1} = l_x - l_x.q_x$ being used as well adapted to a continuous process on a computing machine. The numbers in the $l_x$ column are made integral, the $d_x$ column derived from them and a new $q_x$ formed from $l_x$ and $d_x$ in the usual manner. The $l_x$ column is the one exhibited in Table II. From the $l_x$ and $d_x$ columns the various commutation columns are constructed and the values of the temporary life annuities immediate, last payment at age seventy, set forth in Table VI, are derived by means of the formula:

$$a_{x:\overline{70-x}|} = \frac{N_x - N_{70}}{D_x}. \qquad (16)$$

TABLE XIII.

Commutation Columns, Typhoid Mortality Included, 5 per cent.

| X | D. | N. | M. |
|---|---|---|---|
| 0 .... | 1304688.000 | 19740945.249 | 364642.986 |
| 1 .... | 1055935.235 | 18436257.249 | 178018.225 |
| 2 .... | 963520.1830 | 17380322.014 | 135885.798 |
| 3 .... | 901231.4020 | 16416801.831 | 119478.931 |
| 4 .... | 848706.4497 | 15515570.429 | 109869.766 |
| 5 .... | 801612.3046 | 14666863.979 | 103190.205 |
| 6 .... | 759442.0680 | 13865251.675 | 99191.9833 |
| 7 .... | 719745.3597 | 13105809.607 | 95659.1864 |
| 8 .... | 682365.0765 | 12386064.247 | 92552.4938 |
| 9 .... | 647140.9438 | 11703699.170 | 89821.9304 |
| 10 .... | 613913.2500 | 11056558.226 | 87410.4792 |
| 11 .... | 582528.2549 | 10442644.976 | 85259.4440 |
| 12 .... | 552837.1000 | 9860116.7216 | 83307.7889 |
| 13 .... | 524701.0043 | 9307279.6216 | 81497.2118 |
| 14 .... | 497988.9176 | 8782578.6173 | 79770.8895 |
| 15 .... | 472580.0601 | 8284589.6997 | 78075.7853 |
| 16 .... | 448364.2812 | 7812009.6396 | 76363.8225 |
| 17 .... | 425241.8045 | 7363645.3584 | 74592.0217 |
| 18 .... | 403124.4224 | 6938403.5540 | 72724.2564 |
| 19 .... | 381953.3163 | 6535279.1315 | 70749.5439 |
| 20 .... | 361706.1104 | 6153325.8152 | 68690.5967 |
| 21 .... | 342375.7343 | 5791619.7048 | 66584.3229 |
| 22 .... | 323950.6108 | 5449243.9705 | 64462.8026 |
| 23 .... | 306416.6475 | 5125293.3597 | 62355.0539 |
| 24 .... | 289754.1209 | 4818876.7121 | 60283.8003 |
| 25 .... | 273934.0710 | 4529122.5912 | 58261.5669 |
| 26 .... | 258923.4444 | 4255188.5202 | 56295.4130 |
| 27 .... | 244690.6899 | 3996265.0859 | 54392.3507 |
| 28 .... | 231205.3961 | 3751574.3959 | 52558.9927 |
| 29 .... | 218434.7369 | 3520368.9998 | 50798.1178 |
| 30 .... | 206342.1785 | 3301934.2629 | 49107.2114 |
| 31 .... | 194891.2039 | 3095592.0844 | 47482.0603 |
| 32 .... | 184046.3351 | 2900700.8805 | 45917.7178 |
| 33 .... | 173774.3830 | 2716654.5454 | 44409.8794 |
| 34 .... | 164044.1499 | 2542880.1624 | 42954.6169 |
| 35 .... | 154827.7089 | 2378836.0125 | 41549.7985 |
| 36 .... | 146099.4199 | 2224008.3036 | 40194.2652 |
| 37 .... | 137834.0560 | 2077908.8837 | 38886.0153 |
| 38 .... | 130006.8792 | 1940074.8277 | 37622.3666 |
| 39 .... | 122595.1569 | 1810067.9485 | 36401.4414 |
| 40 .... | 115577.0300 | 1687472.7916 | 35221.1838 |
| 41 .... | 108931.8558 | 1571895.7616 | 34079.6777 |
| 42 .... | 102639.6968 | 1462963.9058 | 32974.7491 |
| 43 .... | 96681.86437 | 1360324.2090 | 31904.5213 |
| 44 .... | 91039.53366 | 1263642.3446 | 30866.0915 |
| 45 .... | 85693.97214 | 1172602.8110 | 29855.7418 |
| 46 .... | 80627.22262 | 1086908.8389 | 28869.6547 |
| 47 .... | 75822.54783 | 1006281.6162 | 27904.3783 |
| 48 .... | 71264.47376 | 930459.06840 | 26956.8979 |
| 49 .... | 66938.34579 | 859194.59464 | 26024.3194 |

| | | | |
|---|---|---|---|
| 50 .... | 62830.72348 | 792256.24885 | 25104.2329 |
| 51 .... | 58929.20683 | 729425.52537 | 24194.6565 |
| 52 .... | 55221.98681 | 670496.31854 | 23293.5908 |
| 53 .... | 51698.20495 | 615274.33173 | 22399.4254 |
| 54 .... | 48348.42341 | 563576.12678 | 21511.4658 |
| 55 .... | 45164.36534 | 515227.70338 | 20629.7136 |
| 56 .... | 42138.90690 | 470063.33804 | 19754.9405 |
| 57 .... | 39264.84322 | 427924.43114 | 18887.4895 |
| 58 .... | 36535.24031 | 388659.58792 | 18027.6438 |
| 59 .... | 33943.57251 | 352124.34760 | 17175.7464 |
| 60 .... | 31483.65042 | 318180.77510 | 16332.1872 |
| 61 .... | 29149.47908 | 286697.12468 | 15497.2370 |
| 62 .... | 26935.67748 | 257547.64560 | 14671.5031 |
| 63 .... | 24837.06174 | 230611.96812 | 13855.5387 |
| 64 .... | 22848.47518 | 205774.90638 | 13049.6691 |
| 65 .... | 20964.17956 | 182926.43120 | 12253.3991 |
| 66 .... | 19178.93007 | 161962.25164 | 11466.4431 |
| 67 .... | 17488.35589 | 142783.32157 | 10689.1491 |
| 68 .... | 15888.59076 | 125294.96568 | 9922.16387 |
| 69 .... | 14377.27054 | 109416.37492 | 9167.44154 |
| 70 .... | 12953.44498 | 95029.104386 | 8428.24851 |
| 71 .... | 11616.71835 | 82075.659402 | 7708.35428 |
| 72 .... | 10366.27147 | 70458.941050 | 7011.08481 |
| 73 .... | 9201.220522 | 60092.669582 | 6339.66535 |
| 74 .... | 8119.862763 | 50891.449060 | 5696.45971 |
| 75 .... | 7119.774720 | 42771.586297 | 5083.03323 |
| 76 .... | 6198.386531 | 35651.811577 | 4500.68141 |
| 77 .... | 5353.485847 | 29453.425046 | 3950.94235 |
| 78 .... | 4582.961868 | 24099 939199 | 3435.34495 |
| 79 .... | 3885.458202 | 19516.977332 | 2956.07933 |
| 80 .... | 3260.216605 | 15631.519130 | 2515 85798 |
| 81 .... | 2705.905762 | 12371.302524 | 2116.79578 |
| 82 .... | 2219.595223 | 9665.3967617 | 1759.33850 |
| 83 .... | 1798.476372 | 7445.8015387 | 1443.91449 |
| 84 .... | 1437.811884 | 5647.3251672 | 1168.89149 |
| 85 .... | 1132.048668 | 4209.5132831 | 931.595544 |
| 86 .... | 875.5580282 | 3077.4646148 | 729.012086 |
| 87 .... | 664.5308142 | 2201.9065866 | 559.678111 |
| 88 .... | 494.6409654 | 1537.3757724 | 421.432653 |
| 89 .... | 359.9092267 | 1042.7348070 | 310.255143 |
| 90 .... | 254.3899707 | 682.82558023 | 221.874540 |
| 91 .... | 172.9920878 | 428.43560956 | 152.590406 |
| 92 .... | 112.1282940 | 255.44352172 | 99.9642613 |
| 93 .... | 68.58879480 | 143.31522772 | 61.7642617 |
| 94 .... | 39.12225086 | 74.72643292 | 35.5638691 |
| 95 .... | 20.46883623 | 35.60418206 | 18.7734060 |
| 96 .... | 9.60379909 | 15.13534583 | 8.88306431 |
| 97 .... | 3.89099230 | 5.53154674 | 3.62758376 |
| 98 .... | 1.29112830 | 1.64055444 | 1.21300616 |
| 99 .... | .31140369 | .34942614 | .29476451 |
| 100 .... | .03802245 | .03802245 | .03621185 |

TABLE XIV.

Commutation Column, Typhoid Motality Excluded, 5 per cent.

| X | D. | N. | M. |
|---|---|---|---|
| 0 .... | 1302659.000 | 19795487.065 | 360016.757 |
| 1 .... | 1054398.093 | 1849282.8065 | 17378.234 |
| 2 .... | 962243.9926 | 17438429.972 | 131842.562 |
| 3 .... | 900169.7456 | 16476185.980 | 115589.458 |
| 4 .... | 847843.4348 | 15576016.234 | 106128.379 |
| 5 .... | 800910.2651 | 14728172.799 | 99568.6984 |
| 6 .... | 758872.7056 | 13927262.534 | 95669.7229 |
| 7 .... | 719313.9761 | 13168389.829 | 92247.7923 |
| 8 .... | 682074.0356 | 12449075.852 | 89260.9002 |
| 9 .... | 646993.9730 | 11767001.817 | 86660.5478 |
| 10 .... | 613913.2500 | 11120007.844 | 84389.0688 |
| 11 .... | 582676.1788 | 10506094.594 | 82385.9575 |
| 12 .... | 553133.8943 | 9923418.4151 | 80590.1569 |
| 13 .... | 525144.8833 | 9370284.5207 | 78940.8575 |
| 14 .... | 498576.3116 | 8845139.6374 | 77379.1874 |
| 15 .... | 473307.3579 | 8346563.3258 | 75851.9581 |
| 16 .... | 449224.6146 | 7873255.9679 | 74307.6641 |
| 17 .... | 426228.2713 | 7424031.3532 | 72702.9649 |
| 18 .... | 404227.6298 | 6997803.0820 | 70998.9147 |
| 19 .... | 383163.8665 | 6593575.4522 | 69184.0788 |
| 20 .... | 363013.1631 | 6210411.5857 | 67279.2794 |
| 21 .... | 343769.1486 | 5847398.4226 | 65321.6077 |
| 22 .... | 325419.5397 | 5503629.2740 | 63341.9551 |
| 23 .... | 307951.0651 | 5178209.7343 | 61369.6441 |
| 24 .... | 291343.5291 | 4870258.6692 | 59426.4485 |
| 25 .... | 275568.2765 | 4578915.1401 | 57524.6987 |
| 26 .... | 260592.0356 | 4303346.8636 | 55670.7598 |
| 27 .... | 246383.4913 | 4042754.8280 | 53871.3548 |
| 28 .... | 232912.9929 | 3796371.3367 | 52133.4018 |
| 29 .... | 220147.9944 | 3563458.3438 | 50459.5017 |
| 30 .... | 208052.5206 | 3343310.3494 | 48847.2636 |
| 31 .... | 196590.6161 | 3135257.8288 | 47292.6276 |
| 32 .... | 185727.7829 | 2938667.2126 | 45791.2450 |
| 33 .... | 175431.1264 | 2752939.4298 | 44338.7712 |
| 34 .... | 165670.5413 | 2577508.3033 | 42932.0493 |
| 35 .... | 156419.2564 | 2411837.7620 | 41569.8340 |
| 36 .... | 147652.6459 | 2255418.5056 | 40251.7673 |
| 37 .... | 139346.2060 | 2107765.8597 | 38976.4046 |
| 38 .... | 131476.3073 | 1968419.6537 | 37742.0412 |
| 39 .... | 124020.5641 | 1836943.3464 | 36547.0676 |
| 40 .... | 116957.7140 | 1712922.7823 | 35389.9635 |
| 41 .... | 110267.4910 | 1595965.0683 | 34269.1554 |
| 42 .... | 103930.5409 | 1485697.5772 | 33183.0375 |
| 43 .... | 97928.17296 | 1381767.0363 | 32129.7429 |
| 44 .... | 92241.56930 | 1283838.8634 | 31106.3882 |
| 45 .... | 86852.12362 | 1191597.2941 | 30109.3941 |
| 46 .... | 81741.88370 | 1104745.1705 | 29134.9666 |
| 47 .... | 76893.82085 | 1023003.2868 | 28179.3814 |
| 48 .... | 72292.32906 | 946109.46590 | 27239.4961 |
| 49 .... | 67922.93251 | 873817.13685 | 26312.5947 |

| | | | |
|---|---|---|---|
| 50 .... | 63772.34936 | 805894.20433 | 25396.4323 |
| 51 .... | 59828.06964 | 742121.85497 | 24488.9321 |
| 52 .... | 56078.75848 | 682293.78533 | 23588.5784 |
| 53 .... | 52513.57535 | 626215.02686 | 22693.8103 |
| 54 .... | 49123.02955 | 573701.45151 | 21803.9136 |
| 55 .... | 45899.07912 | 524578.42195 | 20919.1550 |
| 56 .... | 42834.59978 | 478679.34283 | 20040.3474 |
| 57 .... | 39922.38800 | 435844.74306 | 19167.8766 |
| 58 .... | 37155.63012 | 395922.35506 | 18302.1876 |
| 59 .... | 34527.73073 | 358766.72494 | 17443.6009 |
| 60 .... | 32032.60364 | 324238.99421 | 16692.6538 |
| 61 .... | 29664.33783 | 292206.39057 | 15749.7498 |
| 62 .... | 27417.42437 | 262542.05274 | 14915.4211 |
| 63 .... | 25286.89659 | 235124.62837 | 14090.4849 |
| 64 .... | 23267.37586 | 209837.73178 | 13275.1019 |
| 65 .... | 21353.27511 | 186570.35593 | 12468.9744 |
| 66 .... | 19539.23037 | 165217.08082 | 11671.7516 |
| 67 .... | 17820.84600 | 145677.85045 | 10883.8044 |
| 68 .... | 16194.30503 | 127857.00445 | 10105.8763 |
| 69 .... | 14657.17693 | 111662.69942 | 9339.90388 |
| 70 .... | 13208.61793 | 97005.522488 | 8589.30629 |
| 71 .... | 11848.19006 | 83796.904561 | 7857.86195 |
| 72 .... | 10575.15420 | 71948.714500 | 7149.02598 |
| 73 .... | 9388.658102 | 61373.560298 | 6466.10814 |
| 74 .... | 8287.072434 | 51984.902196 | 5811.60017 |
| 75 .... | 7267.897348 | 43697.829763 | 5187.04904 |
| 76 .... | 6328.640081 | 36429.932415 | 4593.88159 |
| 77 .... | 5467.119452 | 30101.292334 | 4033.72514 |
| 78 .... | 4681.196317 | 24634.172882 | 3508.13969 |
| 79 .... | 3969.565908 | 19952.976564 | 3019.42519 |
| 80 .... | 3331.501876 | 15983.410657 | 2570.38650 |
| 81 .... | 2765.648835 | 12651.908781 | 2163.17664 |
| 82 .... | 2269.081424 | 9886.2599463 | 1798.30741 |
| 83 .... | 1838.982832 | 7617.1785218 | 1476.26014 |
| 84 .... | 1470.529794 | 5778.1956901 | 1195.37746 |
| 85 .... | 1158.070595 | 4307.6658959 | 952.943534 |
| 86 .... | 895.8540150 | 3149.5953009 | 745.873278 |
| 87 .... | 680.0603844 | 2253.7412859 | 572.739362 |
| 88 .... | 506.2763630 | 1573.6809015 | 431.339236 |
| 89 .... | 368.4153208 | 1067.4045384 | 317.586487 |
| 90 .... | 260.4100089 | 698.98921765 | 227.124883 |
| 91 .... | 177.0856677 | 438.57920872 | 156.200958 |
| 92 .... | 114.7798248 | 261.49354106 | 102.327695 |
| 93 .... | 70.21523736 | 146.71371626 | 63.2288715 |
| 94 .... | 40.04960820 | 76.49847890 | 36.4068438 |
| 95 .... | 20.95410973 | 36.44887070 | 19.2184564 |
| 96 .... | 9.82563853 | 15.49476097 | 9.08778865 |
| 97 .... | 3.98782695 | 5.66912244 | 3.71786715 |
| 98 .... | 1.31628015 | 1.68129549 | 1.23621795 |
| 99 .... | .31938840 | .36501534 | .30200688 |
| 100 .... | .04562694 | .04562694 | .04345422 |